

# Calculation of the DC Ionized Field with the Presence of Dielectric Film by FEM and Divergence Theorem

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Corona-generated ionized field on the ground level is one of significant parameters to evaluate the electromagnetic environment of high voltage direct current (HVDC) transmission lines. Greenhouses may be crossed by HVDC power lines due to restricted transmission corridors. Space charges generated by corona discharge of HVDC lines accumulate on the greenhouse films, making the ionized field more complex. In this paper, an algorithm to compute the DC ionized field with the presence of a dielectric film is proposed. Poisson's equation and the current continuity equation are solved by the finite element method (FEM) and the divergence theorem, respectively. The validity of the algorithm is corroborated by the measurement experiments of the DC ionized field.

**Index Terms**—Corona discharge, divergence theorem, FEM, HVDC, ionized field, space charge.

## I. INTRODUCTION

CORONA DISCHARGE occurs when the voltage applied to the electrodes exceeds the inception value. Air maculates are ionized and then ions and ion current are generated. Ionized field strength at ground level is an essential electromagnetic parameter for high voltage direct current (HVDC) transmission lines. Computation and prediction of the ionized field are difficult due to the complicated mechanisms of corona discharge. Because of the limitation of transmission corridors, greenhouses may be crossed by the HVDC lines. Space charges or ions generated from the corona discharge of the HVDC lines accumulate on the greenhouse films, making the ionized field problem more complex.

Finite element method (FEM) is a standard tool to solve the differential equations in many areas in science and engineering. In recent years, the ionized field of HVDC lines has been calculated by FEM [1-2]. But little literature forces on the ionized field with the presence of dielectric films. In this paper, an algorithm to compute the DC ionized field with a dielectric film is presented. Poisson's equation and the current continuity equation are solved by FEM and the divergence theorem.

## II. CALCULATION MODEL

Ionized field vector  $\mathbf{E}$ , ion current density vector  $\mathbf{J}$ , and space charge density  $\rho$  are governed by the following equations [1-2]:

$$\nabla^2 j = -\frac{r}{\varepsilon} \quad (1)$$

$$\nabla \cdot \mathbf{J} = 0 \quad (2)$$

$$\mathbf{J} = K r \mathbf{E} \quad (3)$$

where  $\varphi$  is nodal electric potential, V;  $\varepsilon$  is the electric permittivity of the air,  $8.85 \times 10^{-12} \text{ F}\cdot\text{m}^{-1}$ ;  $K$  is the mobility of the ions,  $1.7 \times 10^{-4} \text{ m}^2\cdot\text{V}^{-1}\cdot\text{s}^{-1}$  and  $2.0 \times 10^{-4} \text{ m}^2\cdot\text{V}^{-1}\cdot\text{s}^{-1}$  for positive and negative ions [2]. The first is Poisson's equation, the second is current continuity equation, and the third is known as Ohm's law in electromagnetics [3].

The configuration of the electrodes is indicated in Fig. 1. A high-voltage conductor with the potential  $U_c$  and radius  $R_c$  is

parallel to the ground. The radius of artificial calculation boundary is  $R_b$ . The surface charge density on the dielectric film is  $\sigma(x)$ . The normal derivative of  $\varphi$ ,  $\partial\varphi/\partial n$ , on the symmetry axis ( $y$  axis) is equal to zero. The electric field strength on the surface of the high-voltage wire remains unchanged after the inception of corona discharge, known as Kaptzov's assumption [4]. It is a two-dimensional (2-D) electric field problem in the rectangular coordinate system.

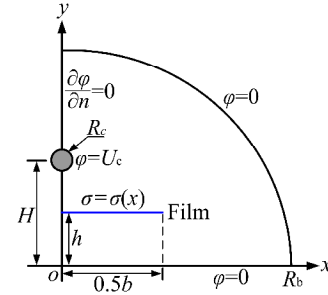


Fig. 1. Calculation region and boundary conditions

## III. ALGORITHM

Poisson's equation is solved by the finite element method (FEM) with triangular elements, as illustrated in Fig. 2. The calculation domain is subdivided by ANSYS into 1649 nodes and 3074 elements, which are read into the procedure written by C programming language. The amplification of the mesh around the wire is shown in Fig. 2(1), which is the black region on the  $y$  axis in Fig. 2(2).

The electric potential  $\varphi$  of each node in the region is calculated by the space charge density  $\rho$  and the boundary conditions in an iteration cycle. The energy functional is written, as [2]

$$F[\mathbf{j}] = \iint_s \frac{\varepsilon}{2} \left[ \left( \frac{\partial j}{\partial x} \right)^2 + \left( \frac{\partial j}{\partial y} \right)^2 \right] dx dy - \iint_s r j dx dy \quad (4)$$

The finite element equation of the electric potential is

$$[\mathbf{K}_t] \{\mathbf{j}\} = \{\mathbf{P}\} \quad (5)$$

where  $[\mathbf{K}_t]$  is the global stiffness matrix,  $\{\varphi\}$  is the column vector of each nodal potential, and  $\{\mathbf{P}\}$  is the column vector calculated from space charge density [2].

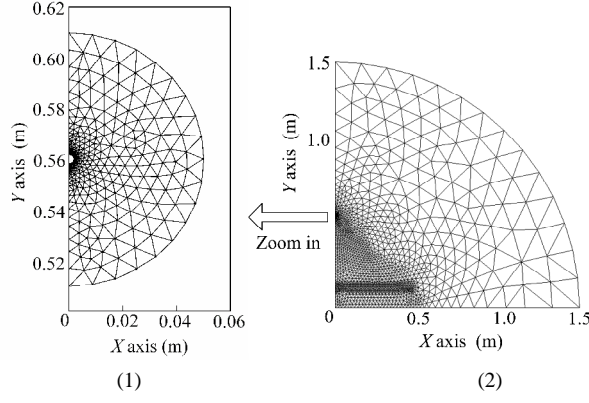


Fig. 2. Meshes of the calculation region to solve the 2-D ionized field problem

The same meshes of calculation region are also used in solving the current continuity equation, which is employed to renew space charge density  $\rho$ .

Substituting (3) into (2), according to the integral statement of the divergence theorem, the equation is written, as

$$\oint_S (K r E) \cdot dS = \int_V \nabla \cdot (K r E) dV = 0 \quad (6)$$

Integral equation (6) is then transformed as an algebraic equation in each triangular element.

$$\sum_{i,j=1,2,3} K(r_{ij} E_{ij} l_{ij}) = 0 \quad (7)$$

where  $\rho_{ij}$  is the average of charge density of  $i$  th and  $j$  th vertices,  $C \cdot m^{-3}$ ;  $E_{ij}$  is the average normal component of the electric field magnitude on the edge  $ij$ ,  $V \cdot m^{-1}$ ;  $l_{ij}$  is the length of edge  $ij$ , m.

The calculation of surface charge density on the dielectric film is based on a 1-D model, as shown in Fig. 3.

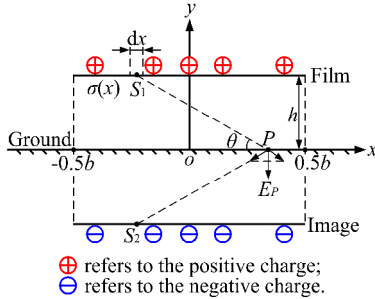


Fig. 3. Calculation of the surface charge density on the dielectric film

Consider a node  $S_1$ , its image  $S_2$ , and a ground-level node  $P$ . The electric field at node  $P$  can be transformed into an algebraic equation and a matrix equation, respectively.

$$E_p = \sum_{i=1}^{N_s} \frac{h \cdot \Delta x}{\pi \epsilon \cdot [(x_i - x_p)^2 + h^2]} \cdot s(x_i) \quad (8)$$

$$[X_{SP}] \{s\} = \{E_p\} \quad (9)$$

where  $N_s$  is the number of the charge sources on the dielectric film, which equals the number of measurement locations on the ground with distance  $\Delta x$ , m;  $\{s\}$  is the column vector of surface charge density on the film;  $\{E_p\}$  is the column vector of the ground-level electric field, which are measured by the field mills; and  $[X_{SP}]$  is the coefficient matrix calculated from the geometric configuration.

## IV. EXPERIMENTS AND RESULTS

A 1.7m-length copper line was used as the HV conductor supported above a grounded aluminum plate by two insulated resin rods with the height 0.56 m. A dielectric film made from polyvinyl chloride was located under the HV conductor. The film dimension and height was 0.75 m $\times$ 0.95 m and 0.12 m.

Measurements of the electric field on the ground plate under the conductor as a function of the lateral distance are shown in Fig. 4. The distribution of the electric field under the unipolar wire is symmetrical. The electric field directly under the conductor is the largest and decreases with the lateral distance from the conductor. The mean relative errors for positive and negative voltage are 6.1% and 5.9%, respectively.

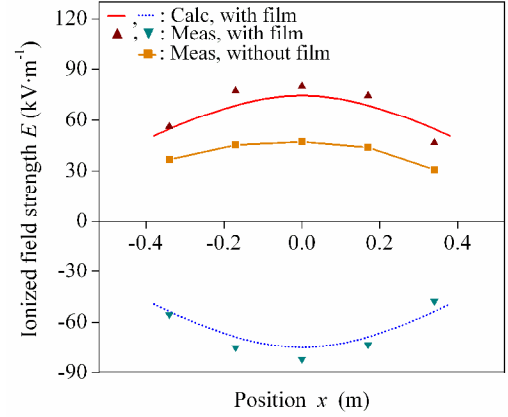


Fig. 4. Distribution of the ground-level electric field strength

## V. CONCLUSIONS

Charges captured by the dielectric film under the HVDC conductor may increase the ground-level electric field strength under the film. An algorithm of the electric field distribution with the presence of the dielectric film is presented by utilizing finite element method and the divergence theorem. The distribution of surface charge density on the film is obtained by the ground-level electric field strength measured by the field mills when the HV conductor is grounded. Relative errors of the ground-level electric field strength between measurements and calculations are approximately 6%.

## ACKNOWLEDGEMENT

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